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Analyzing the Impact of Cash Transfer Policies and Population Disincentives on Income Inequality: An Overlapping Generations Model with Endogenous Fertility

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Content

01

Introduction

A Theoretical Investigation

- Child labour widely studied empirically (Thakurata 2020, Basu 2006, Duflo 2001)
- Theoretical analysis remain limited.
- Focus of this study-
  - Uses an Overlapping Generations (OLG) model with endogenous fertility.
  - Examines how cash transfers and population disincentives shape child labour.
  - Explores the role of parental human capital and wage disparities.
- Key Question:
  - How do policy interventions influence fertility, schooling, and poverty traps?

Parental Dyanmics

- Low human capital parents:
  - Prioritize quantity (more children, less schooling).
  - Depend more on child labour.
- High human capital parents:
  - Invest in quality (fewer children, better education).
  - Prefer adult wages over child wages.

Wage Gap Dynamics

- Large wage gap (Adult wage >> Child Labour Wage):
  - Lower Fertility.
  - More Schooling.
  - Stable Economic Growth.
- Minimal Wage Gap (Adult Wage ≈ Child Labour Wage):
  - Poverty Traps → Dual Equilibria (Rich vs Poor Groups).
  - Persistent Child Labour.

02

Literature Review

Education and Fertility

- There is an inverse relationship between Education and Fertility
  - Becker (1960), Schultz (1997): Higher women's education levels lead to lower fertility rates.
  - Quality-Quantity Trade-off: Investment in child education and health (quality) increases as fertility declines (quantity).
- Morand (1999):
  - Transition from high fertility, low growth to low fertility, high human capital investment.
  - Externalities in human capital can trap economies in low-growth, high-fertility cycles.

Contd.

- Technological Progress and Returns to Education
  - Galor and Weil (2000): Technological progress raises returns to education.
  - Higher education incentives → Lower fertility → Increased income and economic growth.
- Role of Women's Empowerment
  - Prettner and Strulik (2016): Women's empowerment is critical for demographic transitions.
  - More control over reproductive choices → Lower fertility and higher economic participation.

Limitations of Existing Models

- Overlook child labour as a crucial factor in fertility-growth dynamics.
- Child labour sustains high fertility rates by reducing incentives for human capital investment.

Linking Child Labour

- Basu and Van (1998), Baland and Robinson (2000):
  - Provide insights into child labour but do not fully explore its interaction with fertility and human capital.
- Hazan and Berdugo (2002):
  - Technological progress widens wage gaps between adult and child labour → Reduces fertility.

Kitaura & Miyazawa (2023)

- Their findings:
  - Conditional cash transfers (CCTs) help reduce short-term poverty.
  - BUT they may increase fertility in low-income groups, leading to greater income inequality.
  - Did not endogenize schooling time.

Our Paper's Contribution

- Identifies child labour as the root cause of developmental traps and inequality
- We extend the analysis by focusing on population differentials.
  - We identify two key effects on inequality.
    - ➢ Displacement Effect: Changes in steady-state human capital accumulation.
    - ➢ Fertility Differential Effect: Fertility differences between income groups

Four Government Programs

- Key Policy Mechanisms Analyzed:
  - Unconditional Cash Transfers (UCTs).
  - General Conditional Cash Transfers (GCCTs).
  - Special Conditional Cash Transfers (SCCTs)
  - Population Disincentive Policies (PDIPs).

03

The Model

Basic Set up

- Consider a developing economy populated by households consisting of parents and children.
- All households are assumed to be identical.
- The economy is populated by three overlapping generations, which are childhood, adulthood, and old age.
- The parents collectively decide upon their consumption, fertility, and education of their children subject to the budget constraint of the household.
- Absence of Capital Market (Assumption)

- The individuals care about household consumption C_t during adulthood, old-age consumption C_{t+1} , the number of their children n_t ,the human capital of children h_{t+1} , and schooling level s_{t+1} .
- Individual preferences are represented by a simple log-linear utility function -

$$\boldsymbol{U(C_t\ \ C_{t+1}) = \beta ln(C_t) +\ (1-\beta)lnC_{t+1}} \quad \textbf{- - - (1)}$$

where $\beta \in (0,1)$ is the weight given to the present period consumption.

- The children of the household can go to work in the informal sector earn a wage rate $\underline{w}$ and attend school the rest of the time.

- It is assumed that individuals transfer an exogenous fraction b (the norm in society) of their income to their parents.
- It is assumed individuals do not discriminate among their children according to the order of birth or gender or anything else.
- In such a setting, the households face the following budget constraints -

$$\boldsymbol{C_t = (1 - b)\overline{w}h_t(1 - en_t) + \underline{w}(1 - s_t)n_t} \text{ - - - } \textbf{(1)}$$

$$\boldsymbol{C_{t+1} = b\{\overline{w}h_{t+1}(1 - en_{t+1})n_t\}} \text{ - - - } \textbf{(2)}$$

Human Capital Generation

- Incorporated from Galor and Weil (2000) the Human Capital Generation is given by -

$$\boldsymbol{h_{t+1} = \delta(1 + s_t)^{\gamma}} \quad \textbf{- - - - (3)}$$

where $\delta > 0 \ and \ 0 \leq \gamma \leq 1$

- The human capital generation at time t+1 depends on the schooling time, educational technology parameter δ and elasticity of schooling to human capital γ.
- To remove ambiguity, we utilize the classical human capital generation function in the extension, where depends on schooling duration and parental human capital.

Optimum Schooling and Fertility

$$n_t^* = \frac{(1-\beta)(1-\gamma)(1-b)\overline{w}h_t}{(1-b)\overline{w}h_t e \ - \ 2\underline{w}}$$

$$s_t^* = \frac{\gamma(1-b)\overline{w}h_t - \underline{w}(1+\ \gamma)}{\underline{w}(1-\ \gamma)}$$

Proposition 1

- *Child Labour Wage positively impact fertility rate* $\left[\frac{\partial n_t^*}{\partial \underline{w}} > 0\right]$ *and negatively on the level of schooling for a given level of human capital* $\left[\frac{\partial s_t^*}{\partial \underline{w}} < 0\right]$

- *Higher wage differential between parental and child labour lowers the fertility rate* $\left[\frac{\partial n_t^*}{\partial \left(\frac{\overline{w}}{\underline{w}}\right)} < 0\right]$ *and higher the level of schooling* $\left[\frac{\partial s_t^*}{\partial \left(\frac{\overline{w}}{\underline{w}}\right)} > 0\right]$

Three Types of Parents

1) Parents with human capital levels smaller than the threshold $\breve{h}$ will choose not to invest in the education of their children and will encourage them to work in the informal sector with no education. => $s_t^* = 0$ and thus $h_{t+1} = \delta$

2) Parents with human capital levels greater than $\hat{h}$ will not bothered about sending his child to the informal sector and will choose to invest in the education of their children.

=> $s_t^* = 1$ and the children human capital stock $h_{t+1} = \delta 2^{\gamma}$

3) Parents with human capital level in between $\breve{h}$ and $\hat{h}$ will choose to invest some of their time to invest in their children and will also have some elements of child labour.

Optimum Levels for Three Types of Parents

$$s_t^* = \begin{cases} 0 & h_t \leq \breve{h} \\ \dfrac{\gamma(1-b)\overline{w}h_t - \underline{w}(1+\gamma)}{\underline{w}(1-\gamma)} & \breve{h} \leq h_t \leq \hat{h} \\ 1 & h_t \geq \hat{h} \end{cases}$$

Optimum Levels for Three Types of Parents

$$n_t^* = \begin{cases} \dfrac{(1-\beta)(1-\gamma)(1-b)\overline{w}h_t}{(1-b)\overline{w}h_t e \ - \ \underline{w}} & h_t \ \leq \ \check{h} \\ \dfrac{(1-\beta)(1-\gamma)(1-b)\overline{w}h_t}{(1-b)\overline{w}h_t e \ - \ 2\underline{w}} & \check{h} \leq \ h_t \ \leq \ \hat{h} \\ \dfrac{(1-\beta)}{e} & h_t \ \geq \ \hat{h} \end{cases}$$

Dynamical Setting

- Thus, for three kind of parents we have the following dynamical settings.

$$\varphi(h_t) = \begin{cases} \delta & h_t \le \breve{h} \\ \delta\left(\frac{\gamma}{1-\gamma}\right)^{\gamma}\left[\frac{(1-b)\overline{w}h_t e}{\underline{w}} - 2\right]^{\gamma} & \breve{h} \le h_t \le \widehat{h} \\ \delta 2^{\gamma} & h_t \ge \widehat{h} \end{cases}$$

- *where* $\breve{h} = \frac{(1+\gamma)\underline{w}}{\gamma(1-b)\overline{w}e}$ and $\widehat{h} = \frac{2\underline{w}}{\gamma(1-b)\overline{w}e}$ such that $\breve{h} < \widehat{h}$

Contd.

- For, $\check{h} \leq h_t \leq \hat{h}$, $\varphi(h_t)$ is positively sloped and concave. However, for the other two ranges $\varphi(h_t)$ is horizontally straight line.
- The value of h_t cannot be less than δ as indicated by equation (3). Thus, the value of h_t can be either δ or greater than δ.

$$\boldsymbol{h_{t+1} = \delta(1 + s_t)^{\gamma}} \quad \textbf{- - - - (3)}$$

Human Capital Accumulation (When $\breve{h} > \delta$)

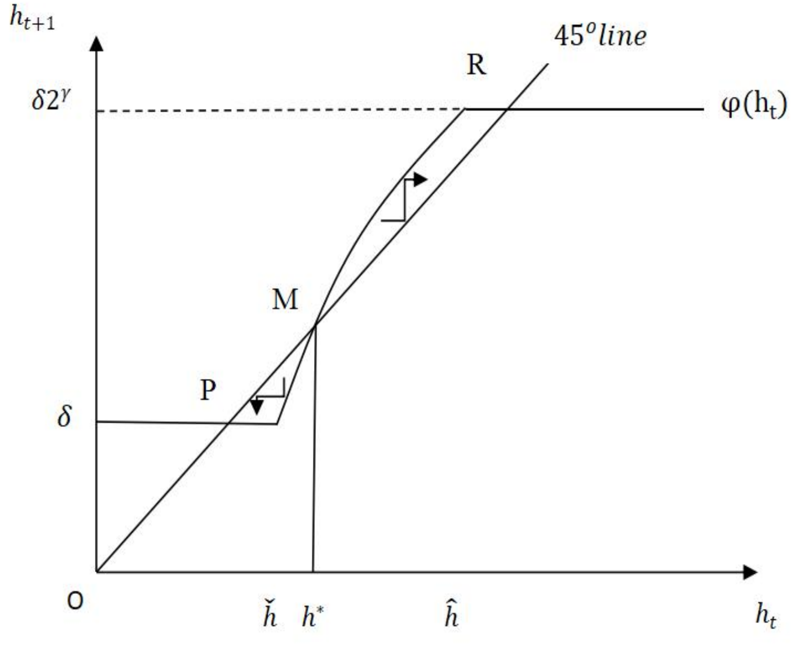

Figure 1

Explanation of Figure - 1

- Parents with human capital between $h^* \; and \; \breve{h}$ converges to P (Bad Equilibrium) and
- Parents with human capital between $h^* \; and \; \hat{h}$ converging to R (Good Equilibrium) .
- Thus, the economy converges to a long-run equilibria where the population is divided into two groups:
  - one group falls into the low level poverty trap and Child labor in such group will increase significantly and
  - the other group will have growing income over time with no child labour.

Human Capital Accumulation (When $\breve{h} = \delta$)

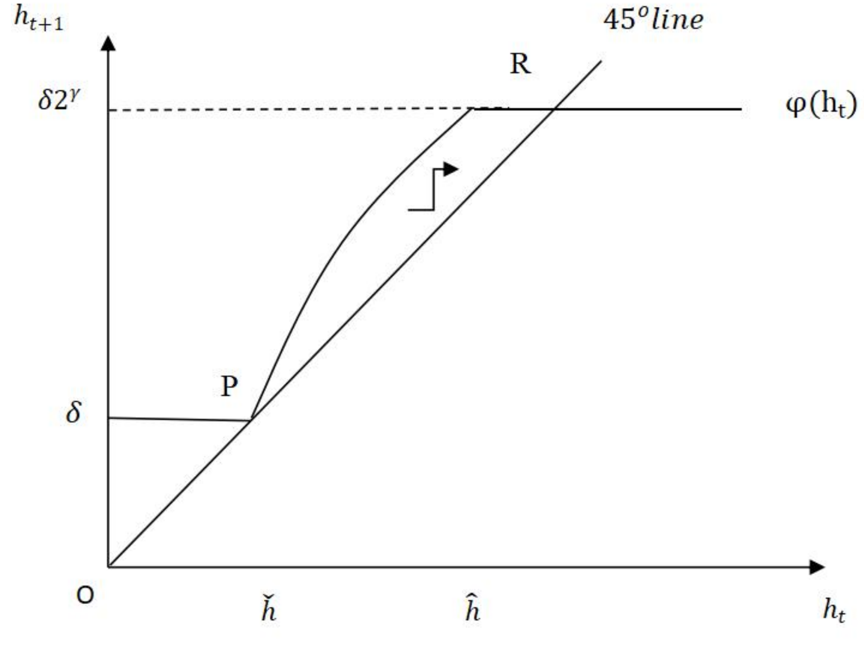

Figure 2

Explanation of Figure - 2

- We have two equilibria P and R .
- But P is unstable, because if human capital is slightly more than the threshold level than it will eventually converge to good equilibrium.
- Hence, we have only one stable equilibrium here and every one converging to this good equilibrium eventually.

Condition for Developmental Trap

- The lower threshold level of human capital will be greater than educational technology parameter if the following condition holds - $\breve{h} < \delta$. This implies,

$$\breve{\boldsymbol{h}} = \frac{(\mathbf{1}+\boldsymbol{\gamma})\underline{\boldsymbol{w}}}{\boldsymbol{\gamma}(\mathbf{1}-\boldsymbol{b})\overline{\boldsymbol{w}}\boldsymbol{e}} \leq \boldsymbol{\delta} \Leftrightarrow \frac{\overline{\boldsymbol{w}}}{\underline{\boldsymbol{w}}} < \frac{\mathbf{1}+\boldsymbol{\gamma}}{\boldsymbol{\delta\gamma}(\mathbf{1}-\boldsymbol{b})\boldsymbol{e}} \text{ - - - } (*)$$

Proposition 2

- *If the Wage Differential Between Parents and Child Labour is low, which can be the case, when adult wage is low and (or) child labour wage is high,*
  - *then the economy converges to dual equilibrium with poor and rich.*
  - *The Group of Poor will be converged to the poverty trap.*

$$\frac{\overline{w}}{\underline{w}} < \frac{1+\gamma}{\delta\gamma(1-b)e} \text{ - - - } (*)$$

Contd.

Otherwise, if the wage differential between parents and child labour is high, when adult wage is high and (or) child labour wage is low, then in the long run the economy converges to a unique stable equilibrium where we will have zero child labour and full schooling, and ultimately no developmental trap.

$$\frac{\overline{w}}{\underline{w}} = \frac{1+\gamma}{\delta\gamma(1-b)e} \text{ - - - } (*)'$$

Lemma 1

- *The condition for developmental trap diminishes with*
  - *increase in educational technology* δ
  - *increase in child Bearing cost e,*
  - *sound pension system from Public and Private Institutions* $(1-b)$,
  - *higher elasticity of human capital with respect to schooling* γ *and*
  - *partial ban on child labour.* $\underline{w}$

$$\frac{\overline{\boldsymbol{w}}}{\underline{\boldsymbol{w}}} < \frac{\mathbf{1}+\boldsymbol{\gamma}}{\boldsymbol{\delta\gamma}(\mathbf{1}-\boldsymbol{b})\boldsymbol{e}} \text{ - - - } (*)$$

04

Government Programs

Cash Transfer Program

- Suppose that government is assumed to adopt the following Cash Transfer (CT) program of the form :- $\boldsymbol{CT = T + \alpha \underline{w} n_t s_t}$
- Where T is is the transfer that is independent of the level of schooling.
- $\alpha \underline{w} n_t s_t$ is the kind of transfer which is dependent on the level of schooling of the children and α is the rate of education subsidy.
- When $\alpha = 0$, the CT program is called "Unconditional Cash Transfer" and when $\alpha > 0$, this program is called as "General Conditional Cash Transfer" as it is conditioned on the level of schooling of the children concerned.

Government Budget Constraint

- We assume that the governments of poor under developing countries are supported by
  - development banks
  - and other international development bodies
  - such that this cash transfer program are fully sponsored and supported by foreign aid.
    - In this case, the government budget constraint is given by -

$$\boldsymbol{A_t = \int CT_t dF(h_t)}$$

Unconditional Cash Transfer (UCT)

- Unconditional Cash Transfer Programs are designed
  - to provide financial assistance to individuals or households
  - without requiring specific actions or conditions to be met (such as enrolling children in school or getting regular health checkups,
  - which are typically required in CCT programs).
    - In such a setting, we can rewrite the household budget constraints as -

$$\boldsymbol{C_t = (1-b)\overline{w}h_t(1-en_t) + \underline{w}(1-s_t)n_t + T} \text{ - - - } \textbf{(1.1)}$$

$$\boldsymbol{C_{t+1} = b\{\overline{w}h_{t+1}(1-en_{t+1})n_t\}} \text{ - - - } \textbf{(2.1)}$$

Optimal Schooling and Fertility

$$n_t^* = \frac{(1-\beta)(1-\gamma)\{(1-b)\overline{w}h_t + T\}}{(1-b)\overline{w}h_t e - 2\underline{w}}$$

$$s_t^* = \frac{\gamma(1-b)\overline{w}h_t - \underline{w}(1+\gamma)}{\underline{w}(1-\gamma)}$$

- Note that there is no unconditional cash transfer (T) term on schooling, that is,
  - there is no UCT effect on schooling and fertility is increasing.

Optimum Levels for Three Types of Parents

$$s_t^{*UCT} = \begin{cases} 0 & h_t \le \check{h}_{UCT} \\ \dfrac{\gamma(1-b)\overline{w}h_t e - \underline{w}(1+\gamma)}{\underline{w}(1-\gamma)} & \hat{h}_{UCT} \le h_t \le \check{h}_{UCT} \\ 1 & h_t \ge \hat{h}_{UCT} \end{cases}$$

Optimum Levels for Three Types of Parents

$$n_t^{*UCT} = \begin{cases} \dfrac{(1-\beta)\{(1-b)\overline{w}h_t + T\}}{(1-b)\overline{w}h_t e - \underline{w}} & h_t \le \check{h}_{UCT} \\ \dfrac{(1-\beta)(1-\gamma)\{(1-b)\overline{w}h_t + T\}}{(1-b)\overline{w}h_t e - 2\underline{w}} & \hat{h}_{UCT} \le h_t \le \check{h}_{UCT} \\ \dfrac{(1-\beta)\{(1-b)\overline{w}h_t + T\}}{(1-b)\overline{w}h_t e} & h_t \ge \hat{h}_{UCT} \end{cases}$$

No Change in the Threshold Level

$$\breve{h}_{UCT} = \breve{h} = \frac{(1+\gamma)\underline{w}}{\gamma(1-b)\overline{w}e} \quad \text{and} \quad \hat{h}_{UCT} = \hat{h} = \frac{2\underline{w}}{\gamma(1-b)\overline{w}e}$$

$$\text{such that } \breve{h}_{UCT} < \hat{h}_{UCT}$$

– *No Change in the threshold level of Human Capital due to Unconditional Cash Transfer.*

Dynamical Setting under UCT

- Thus, for three kind of parents we have the following dynamical settings.

$$\varphi(h_t)^{UCT} = \begin{cases} \delta & h_t \le \check{h}_{UCT} \\ \delta\left(\frac{\gamma}{1-\gamma}\right)^{\gamma}\left[\frac{(1-b)\overline{w}h_t e}{\underline{w}} - 2\right]^{\gamma} & \hat{h}_{UCT} \le h_t \le \check{h}_{UCT} \\ \delta 2^{\gamma} & h_t \ge \hat{h}_{UCT} \end{cases}$$

Note that $\varphi(h_t) = \varphi(h_t)^{UCT}$

Proposition 3

- *UCTs only increases the fertility rate to all the three groups without any change in the level of schooling .*
- *The human capital generation and the upper and lower threshold levels of human capital is unaffected by UCTs.*

General Conditional Cash Transfer (GCCT)

- It is a form of social assistance program designed to alleviate poverty by providing financial aid contingent upon certain behavioral requirements.
- Thus, in Case of Conditional cash transfer (T=0).

$$\boldsymbol{CT = \alpha \underline{w} n_t s_t}$$

  - In such a setting, we can rewrite the household budget constraints as -

$$\boldsymbol{C_t = (1-b)\overline{w}h_t(1-en_t) + \underline{w}(1-s_t)n_t + \alpha \underline{w} n_t s_t} \text{ - - - } \mathbf{(1.2)}$$

$$\boldsymbol{C_{t+1} = b\{\overline{w}h_{t+1}(1-en_{t+1})n_t\}} \text{ - - - } \mathbf{(2.2)}$$

Optimal Schooling and Fertility

$$n_t^* = \frac{(1-\beta)(1-\gamma)\{(1-b)\overline{w}h_t\}}{(1-b)\overline{w}h_t e \; - \; \underline{w}(2-\alpha)}$$

$$s_t^* = \frac{\gamma(1-b)\overline{w}h_t - \underline{w}\{\gamma \; + \; (1-\alpha)\}}{\underline{w}(1- \; \gamma)(1-\alpha)}$$

- Unlike UCT, GCCT have a negative effect on fertility rates $\left[\frac{\partial n_t^*}{\partial \alpha} \; < 0\right]$ but a positive effect on educational attainment $\left[\frac{\partial s_t^*}{\partial \alpha} \; > \; 0\right]$ (See Appendix 4)

Optimal Schooling For Different Human Capital

$$
s_t^* = \begin{cases} 0 & h_t \leq \check{h}_{GCCT} \\ \dfrac{\gamma(1-b)\overline{w}h_t - \underline{w}\{\gamma + (1-\alpha)\}}{\underline{w}(1-\gamma)(1-\alpha)} & \check{h}_{GCCT} \leq h_t \leq \hat{h}_{GCCT} \\ 1 & h_t \geq \hat{h}_{GCCT} \end{cases}
$$

Optimal Fertility For Different Human Capital

$$\boldsymbol{n_t^*} = \begin{cases} \dfrac{(1-\beta)\{(1-b)\overline{w}h_t\}}{(1-b)\overline{w}h_t e - \underline{w}} & h_t \le \check{h}_{GCCT} \\ \dfrac{(1-\beta)(1-\gamma)\{(1-b)\overline{w}h_t\}}{(1-b)\overline{w}h_t e - \underline{w}(2-\alpha)} & \check{h}_{GCCT} \le h_t \le \hat{h}_{GCCT} \\ \dfrac{(1-\beta)\{(1-b)\overline{w}h_t\}}{(1-b)\overline{w}h_t e - \underline{w}\alpha} & h_t \ge \hat{h}_{GCCT} \end{cases}$$

Dynamical Setting under GCCT

$$\varphi(h_t)^{UCT} = \begin{cases} \delta & h_t \le \check{h}_{GCCT} \\ \delta\left(\frac{\gamma}{1-\gamma}\right)^{\gamma}\left[\frac{(1-b)\overline{w}h_t e}{\underline{w}(1-\alpha)} - \frac{2-\alpha}{1-\alpha}\right]^{\gamma} & \check{h}_{GCCT} \le h_t \le \hat{h}_{GCCT} \\ \delta 2^{\gamma} & h_t \ge \hat{h}_{GCCT} \end{cases}$$

Where,

$\check{h}_{GCCT} = \frac{\{(1-\alpha)+\gamma\}\underline{\underline{w}}}{\gamma(1-b)\overline{w}e} \le \check{h}$ and $\hat{h}_{GCCT} = \frac{\{2-\alpha(2-\gamma)\underline{\underline{w}}\}}{\gamma(1-b)\overline{w}e} \le \hat{h}$ such that $\check{h}_{GCCT} < \hat{h}_{GCCT}$

Contd.

- As both high and low threshold level of human capital is shifting leftward it leads to a fall in the steady state level of human capital accumulation.
- This suggests that less number of people are now in the low - equilibrium group and consequently some people are getting the big - push to converge towards the high equilibrium, if not all.

Dynamics of Human Capital under GCCT

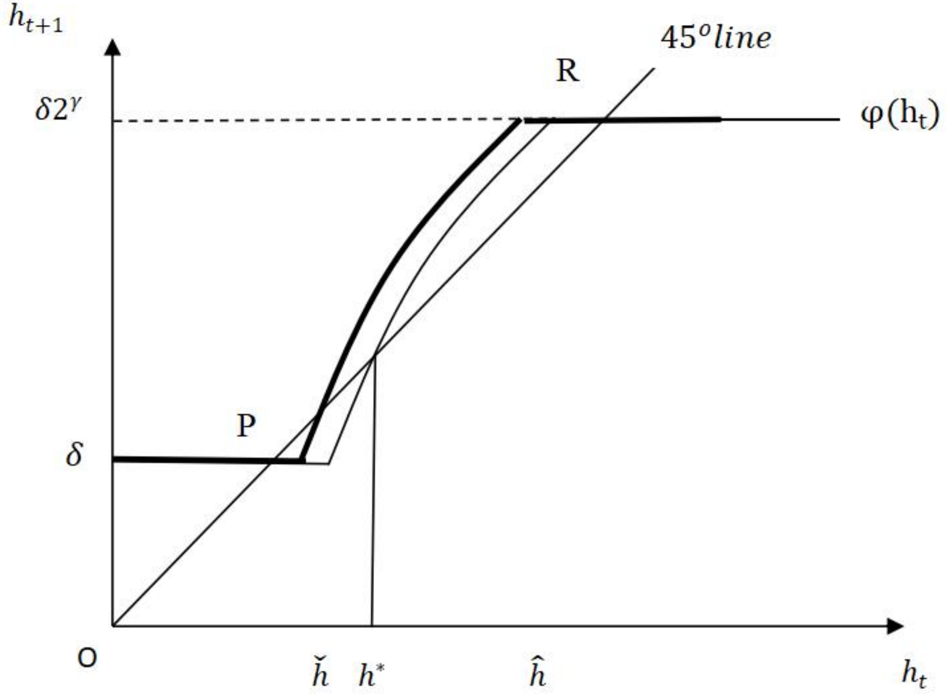

Figure - 3

Explanation of Figure - 3

- In the long run, General Conditional Cash Transfer,
  - act as a catalyst to remove the developmental traps to a certain possible extent,
    - due to a positive displacement effect.
- The positive displacement effect is defined as the fall in the steady state value.
  - This indicates that less number of people are now in the low - equilibrium group.
    - And consequently some people are getting the big - push to converge towards the high equilibrium.

Special Conditional Cash Transfer (SCCTs)

- Let us suppose that under conditional cash transfer in addition to kind of transfer which which was set
  - with the objective of compensating for the opportunity cost of the children's school attendance ,
  - we add one more constraint where the transfer is provided to only those children who reach some threshold level $\bar{s}$.
- From the above assumption we can rewrite the household budget constraint (1) as -

$$\boldsymbol{C_t = (1-b)\overline{w}h_t(1-en_t) + \underline{w}(1-s_t)n_t + \alpha\underline{w}n_t(s_t - \bar{s})} \quad \text{---} \ \mathbf{(1.3)}$$

Optimal Schooling and Fertility under SCCT

$$\boldsymbol{n_t^{*SCCT}} = \frac{(\mathbf{1}-\boldsymbol{\beta})(\mathbf{1}-\boldsymbol{\gamma})\{(\mathbf{1}-\boldsymbol{b})\overline{\boldsymbol{w}}\boldsymbol{h_t}\}}{(\mathbf{1}-\boldsymbol{b})\overline{\boldsymbol{w}}\boldsymbol{h_t}\boldsymbol{e} - \underline{\boldsymbol{w}}\{\mathbf{2}-\boldsymbol{\alpha}(\mathbf{1}+\overline{\boldsymbol{s}})\}}$$

$$\boldsymbol{s_t^{*SCCT}} = \frac{\boldsymbol{\gamma}(\mathbf{1}-\boldsymbol{b})\overline{\boldsymbol{w}}\boldsymbol{h_t} - \underline{\boldsymbol{w}}\{\boldsymbol{\gamma}+\mathbf{1}-\boldsymbol{\alpha}(\mathbf{1}+\boldsymbol{\gamma}\overline{\boldsymbol{s}})\}}{\underline{\boldsymbol{w}}(\mathbf{1}-\boldsymbol{\gamma})(\mathbf{1}-\boldsymbol{\alpha})}$$

- The fertility rate is falling and school time is increasing (Appendix 5)
  - Not only compensating for the opportunity cost of school's attendance but also improvising a threshold can substantially have positive impacts in terms of lower fertility rate and higher schooling.

One More Constraint

- Now those who have the level of schooling less than won't receive any such benefits, so,

$$s_t^* = \frac{\gamma(1-b)\overline{w}h_t - \underline{w}(1+\gamma)}{\underline{w}(1-\gamma)} > \overline{s}$$

$$h_t > \frac{\underline{w}\{1+\gamma+\overline{s}(1-\gamma)\}}{\gamma(1-b)\overline{w}e} = \widetilde{h} \text{ where } \breve{h} \leq \widetilde{h} \leq \widehat{h}$$

Four Types of Parents

- Here, thus we have four types of parents with respect to their human capital accumulation -

1. Parents with human capital below $\check{h}$ may prioritize sending their children to work in the informal sector to earn money, rather than focusing on education.

2. Parents with human capital between $\check{h}$ and $\tilde{h}$ will choose s_t^* and n_t^*

3. Parents with human capital between $\tilde{h}$ and $\hat{h}$ will choose ${s_t^*}^{SCCT}$ and ${n_t^*}^{SCCT}$

4. Parents with human capital above $\hat{h}$, prioritize education over child labour and invest all their children's time in education.

Optimal Schooling For Different Human Capital

$$
s_t^* = \begin{cases} 0 & h_t \le \breve{h} \\ \dfrac{\gamma(1-b)\overline{w}h_t - \underline{w}(1+\gamma)}{\underline{w}(1-\gamma)} & \breve{h} \le h_t \le \tilde{h} \\ \dfrac{\gamma(1-b)\overline{w}h_t - \underline{w}\{\gamma + 1 - \alpha(1+\gamma\bar{s})\}}{\underline{w}(1-\gamma)(1-\alpha)} & \tilde{h} \le h_t \le \hat{h} \\ 1 & h_t \ge \hat{h} \end{cases}
$$

Optimal Fertility For Different Human Capital

$$
n_t^* = \begin{cases}
\dfrac{(1-\beta)(h_t 1-\gamma)\{(1-b)\overline{w}h_t\}}{(1-b)\overline{w}e - \underline{w}} & h_t \le \breve{h} \\[2ex]
\dfrac{(1-\beta)(h_t 1-\gamma)\{(1-b)\overline{w}h_t\}}{(1-b)\overline{w}e - 2\underline{w}} & \breve{h} \le h_t \le \tilde{h} \\[2ex]
\dfrac{(1-\beta)(1-\gamma)\{(1-b)\overline{w}h_t\}}{(1-b)\overline{w}h_t e - \underline{w}\{2-\alpha(1+\overline{s})\}} & \tilde{h} \le h_t \le \hat{h} \\[2ex]
\dfrac{1-\beta}{e} & h_t \ge \hat{h}
\end{cases}
$$

Dynamical System under SCCT

$$\boldsymbol{\varphi(h_t)^{SCCT}} = \begin{cases} \boldsymbol{\delta} & \boldsymbol{h_t \le \check{h}} \\ \boldsymbol{\delta\left(\frac{\gamma}{1-\gamma}\right)^{\gamma}\left[\frac{(1-b)\overline{w}h_t e}{\underline{w}(1-\alpha)} - 2\right]^{\gamma}} & \boldsymbol{\check{h} \le h_t \le \tilde{h}} \\ \boldsymbol{\delta\left(\frac{\gamma}{1-\gamma}\right)^{\gamma}\left[\frac{(1-b)\overline{w}h_t e}{\underline{w}(1-\alpha)} - \frac{2-\alpha}{1-\alpha}\right]^{\gamma}} & \boldsymbol{\tilde{h} \le h_t \le \hat{h}} \\ \boldsymbol{\delta 2^{\gamma}} & \boldsymbol{h_t \ge \hat{h}} \end{cases}$$

Contd.

- From Appendix 7 we can see that at $\tilde{h}$,
  - $\varphi(h_t)^{SCCT}$ for $\check{h} \leq h_t \leq \tilde{h}$ is less than $\varphi(h_t)^{SCCT}$ for $\tilde{h} \leq h_t \leq \hat{h}$,
  - indicates that there is a discontinuity at $\tilde{h}$.
  - And at $\hat{h}$, $\varphi(h_t)^{SCCT}$ for $h_t \geq \hat{h}$ is greater than $\varphi(h_t)^{SCCT}$ for $\tilde{h} \leq h_t \leq \hat{h}$ which again leads to another discontinuity.

Dynamics of Human Capital under SCCT

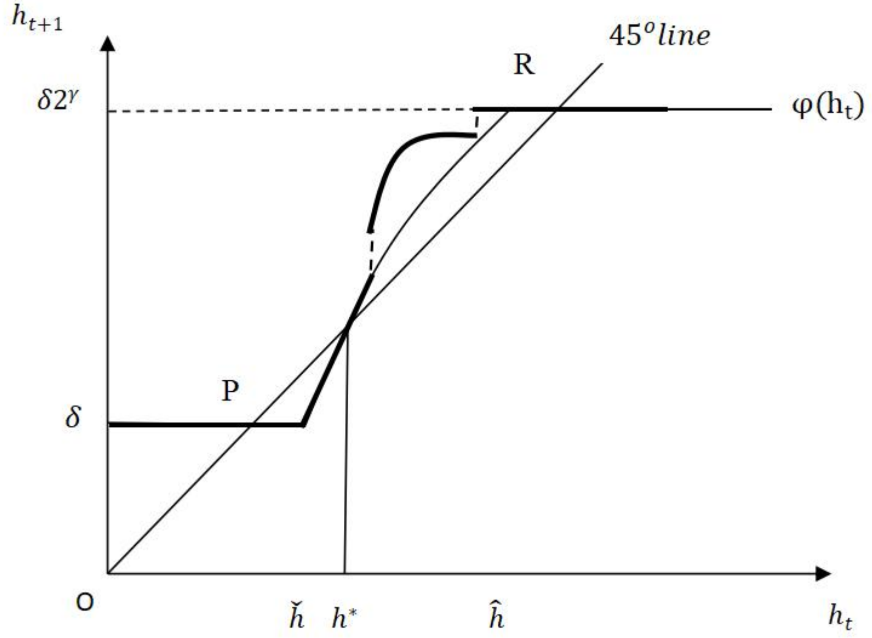

Figure - 4

Explanation of Figure 4 - Proposition 4

- *The Special Conditional Cash Transfer has no change in the unstable steady state, and thus,*
  - *there exists no displacement effect*
  - *and subsequently no change in the poverty trap condition with such transfers.*

Population Dis-incentive Program (PDIP)

- Under this program, households that choose to have more than two children are required to bear a cost.
- The government is assumed to adopt the following dis-incentive program of the form:-

$$P = \begin{cases} \overline{e}(n_t - 2)\overline{w}h_t & n_t > 2 \\ 0 & n_t \leq 2 \end{cases}$$

- Where, P is the Penalty charged on the Parent's income for more than two children.
- From the above assumption we can rewrite the household budget constraint as -

$$\boldsymbol{C_t = (1-b)\{\overline{w}h_t(1-en_t) - \overline{e}(n_t-2)\overline{w}h_t\} + \underline{w}(1-s_t)n_t} \text{ - - - } \mathbf{(1.4)}$$

Optimal Schooling and Fertility

$$n_t^{*PDIP} = \frac{(1-\beta)(1-\gamma)(1-b)\overline{w}h_t(1+2\overline{e})}{(1-b)\overline{w}h_t(e+\overline{e}) - 2\underline{w}}$$

$$s_t^{*PDIP} = \frac{\gamma(1-b)\overline{w}h_t(e+\overline{e}) - \underline{w}(1+\gamma)}{\underline{w}(1-\gamma)}$$

- Here, $\frac{dn_t^{*PDIP}}{d\overline{e}} < 0 \; and \; \frac{ds_t^{*PDIP}}{d\overline{e}} > 0$, this readily asserts that population dis - incentive program decreases fertility rate and increases the schooling level.

Optimal Schooling For Different Human Capital

$$\boldsymbol{s_t^*} = \begin{cases} \boldsymbol{0} & \boldsymbol{h_t \leq \check{h}_{PDIP}} \\ \dfrac{\boldsymbol{\gamma(1-b)\overline{w}h_t(e+\overline{e}) - \underline{w}(1+\gamma)}}{\boldsymbol{\underline{w}(1-\gamma)}} & \boldsymbol{\check{h}_{PDIP} \leq h_t \leq \hat{h}_{PDIP}} \\ \boldsymbol{1} & \boldsymbol{h_t \geq \hat{h}_{PDIP}} \end{cases}$$

Optimal Fertility For Different Human Capital

$$\boldsymbol{n_t^*} = \begin{cases} \dfrac{(1-\beta)(1-b)\overline{w}h_t(1+2\overline{e})}{(1-b)\overline{w}h_t(e+\overline{e}) - \underline{w}} & h_t \le \check{h}_{PDIP} \\[2ex] \dfrac{(1-\beta)(1-\gamma)(1-b)\overline{w}h_t(1+2\overline{e})}{(1-b)\overline{w}h_t(e+\overline{e}) - 2\underline{w}} & \check{h}_{PDIP} \le h_t \le \hat{h}_{PDIP} \\[2ex] \dfrac{(1-\beta)(1-b)\overline{w}h_t(1+2\overline{e})}{(1-b)\overline{w}h_t(e+\overline{e})} & h_t \ge \hat{h}_{PDIP} \end{cases}$$

Contd.

$$\breve{h}_{PDIP} = \frac{\underline{w}(1+\gamma)}{\gamma(1-b)\overline{w}(e+\overline{e})} \leq \breve{h} \quad \text{and} \quad \hat{h}_{PDIP} = \frac{2\underline{w}}{\gamma(1-b)\overline{w}(e+\overline{e})} \leq \hat{h}$$

such that $\breve{h}_{PDIP} < \hat{h}_{PDIP}$

- As both high and low threshold level of human capital is shifting leftward it leads to a fall in the level of h^*
  - which suggests some poor people are coming out of the developmental trap

Dynamical Setting Under PDIP

- $$\boldsymbol{\varphi(h_t)^{PDIP}} = \begin{cases} \boldsymbol{\delta} & \boldsymbol{h_t \le \check{h}_{PDIP}} \\ \boldsymbol{\delta\left(\frac{\gamma}{1-\gamma}\right)^{\gamma}\left[\frac{\gamma(1-b)\overline{w}h_t(e+\overline{e})-2\underline{w}}{\underline{w}}\right]^{\gamma}} & \boldsymbol{\check{h}_{PDIP} \le h_t \le \hat{h}_{PDIP}} \\ \boldsymbol{\delta 2^{\gamma}} & \boldsymbol{h_t \ge \hat{h}_{PDIP}} \end{cases}$$
- Thus, the positive displacement effect reduces the population differential between the rich and the poor partially.

Population Differential Effect

- We define the population differential between the two groups with:

$$\mu_t = \frac{N^P}{N^R} = \frac{N_0^P}{N_0^R}\left(\frac{n_{t+1}^P}{n_{t+1}^R}\right)^t$$

Where $N_0^P = \int_{\delta}^{h^*} \frac{\gamma\delta^\gamma}{H^{\gamma+1}} dh = 1 - \left(\frac{\delta}{h^*}\right)^\gamma$ and $N_0^R = \left(\frac{\delta}{h^*}\right)^\gamma$

Taken From Pareto Distribution.

Pareto Distribution

- If human capital accumulation follows a Pareto Distribution with the density function:-

$$f(\mathrm{x}) \ = \ \frac{\lambda m^{\lambda}}{X^{\lambda+1}};\ x \geq \ m\,,\lambda \ > \ 0$$

where γ is the Pareto inequality parameter.

- The cumulative density function is given by -

$$F(x) \ \int_{m}^{x} \frac{\lambda m^{\lambda}}{X^{\lambda+1}} dX \ = \ 1 - \ \left(\frac{m}{x}\right)^{\lambda}$$

Population Differential in Basic Set up

- In the Basic Set up of the model without government intervention, we have,

$$\mu_t = \left[\left(\frac{h^*}{\delta}\right)^{\gamma} - 1\right]\left[\frac{(1-\mathrm{b})\overline{\mathrm{w}}\mathrm{h_t e}}{(1-\mathrm{b})\overline{\mathrm{w}}\mathrm{h_t e} - \underline{\mathrm{w}}}\right]^t$$

- Here, change in h^* indicates Diplacement Effect. (Short Term Effect)

- And the term $\frac{(1-\mathrm{b})\overline{\mathrm{w}}\mathrm{h_t e}}{(1-\mathrm{b})\overline{\mathrm{w}}\mathrm{h_t e}-\underline{\mathrm{w}}}$ indicates Fertility Differential Effect. (Long Term Effect)

Population Differential in case of UCT

- In case of unconditional cash transfer

$$\mu_t = \left[\left(\frac{h^*}{\delta}\right)^{\gamma} - 1\right]\left[\frac{(1-\mathrm{b})\overline{\mathrm{w}}\mathrm{h_t e}}{(1-\mathrm{b})\overline{\mathrm{w}}\mathrm{h_t e} - \underline{\mathrm{w}}}\right]^t$$

- As h^* is same and there is no change in the fertility differential in case of UCTs.
  - The population differential between the two groups remain same as in the standard model.
  - This indicates that there is no impact on inequality due to such transfers.

Population Differential in case of GCCT

- In the long run, it decreases the fertility rate in rich group section with no effect in the poor group section.
- Hence, fertility differential is decreasing with increase in the rate of education subsidy, this is purely positive fertility differential effect.
- We can also see that the steady state value h^* is decreasing as indicated by figure 3, which implies a positive displacement effect.

$$\mu_t = \left[\left(\frac{h^*}{\delta}\right)^{\gamma} - 1\right]\left[\frac{(1-\mathrm{b})\overline{\mathrm{w}}\mathrm{h_t e} - \mathrm{w}\alpha}{(1-\mathrm{b})\overline{\mathrm{w}}\mathrm{h_t e} - \underline{\mathrm{w}}}\right]^t$$

Population Differential in case of SCCT

- In this case, the fertility differential effect is rising with a rise in the critical level of schooling above which children will get such cash transfer, with no change in the low steady state value of h^*.
- Thus, a negative fertility differential effect with zero displacement effect.

$$\mu_t = \left[\left(\frac{h^*}{\delta}\right)^{\gamma} - 1\right]\left[1 + \frac{\underline{w}\alpha(1 + \bar{s})}{(1-b)\overline{w}h_t e - 2\underline{w}}\right]^t$$

- SCCT is less effective than GCCT in eliminating Inequality in the Long Run, but in the short run it increases schooling and decreases fertility.

Population Differential in Case of PDIP

- In case of Population Disincentives Program, due to fall in the steady state level of human capital accumulation, there is a positive displacement effect.
- Additionally, the fertility differential is decreasing with increase in such penalty. (See Appendix 6).

$$\mu_t = \left[\left(\frac{h^*}{\delta}\right)^{\gamma} - 1\right]\left[\frac{(1-\mathrm{b})\overline{\mathrm{w}}\mathrm{h_t}(\mathrm{e}+\overline{e})}{(1-\mathrm{b})\overline{\mathrm{w}}\mathrm{h_t}(\mathrm{e}+\overline{e})-\underline{\mathrm{w}}}\right]^{t}$$

Contd.

- As the population differential is decreasing without any condition but the poor and
  - rich steady state population is decreasing if e < 1/2,
  - indicates that decrease in the numerator must be strongly greater than decrease in the numerator of .
  - That is,
    - The fall in the fertility rate is the low income group is stronger than the fall in the fertility rate in the high income group.

Proposition 5

- *Unconditional Cash Transfers (UCTs)*
  - *do not influence fertility differentials*
  - *nor generate displacement effects,*
  - *resulting in no impact on inequality.*
- *In contrast, General Conditional Cash Transfers (GCCTs),*
  - *positively affect both fertility differential*
  - *and displacement effects,*
  - *leading to a reduction in inequality.*

Contd.

- *However, Special Conditional Cash Transfers (SCCTs)*
  - *negatively impact fertility differentials*
  - *without altering displacement effects,*
  - *ultimately increasing inequality.*
- *On the other hand, Population Disincentive Programs (PDIPs)*
  - *positively influence both fertility and displacement effects.*

Table 1 - Summarizing the Result of Proposition 5

	Fertility	Schooling	Inequality
UCT	+	0	0
GCCT	-	+	-
SCCT	-	+	+
PDIP	-	+	-

05

Extention

Incorporating Parent's Human Capital in Human Capital Accumulation.

- Let us assume that the human capital generation is -

$$h_{t+1} = \delta h_t^{1-\gamma} s_t^{\gamma}$$

where $\delta > 0$ and $0 < \gamma < 1$

- We added h_t to capture the significant influence of parent's human capital on their children's human capital.

Dynamical Setup

- In the basic setup of the model, for three kinds of parent, the dynamical system will be -

$$\boldsymbol{\varphi(h_t) = \begin{cases} \delta {h_t}^{1-\gamma} & h_t \le \breve{h} \\ \delta\left(\frac{\gamma}{1-\gamma}\right)^{\gamma} {h_t}^{1-\gamma}\left[\frac{(1-b)\overline{w}h_t e}{\underline{w}} - 2\right]^{\gamma} & \breve{h} \le h_t \le \widehat{h} \\ \delta {h_t}^{1-\gamma}\, 2^{\gamma} & h_t \ge \widehat{h} \end{cases}}$$

Explantion of New Dynamical Setup

- For $h_t \leq \check{h}$, and for $h_t \geq \hat{h}$, for we have $\varphi'(h_t) > 0$, i.e.,
  - in both these ranges the function is positively sloped and concave
  - and the function in between the two threshold level is also positively sloped and concave (see appendix 7).
  - However, the slopes are different at $\hat{h}$ for the two different functional forms of the dynamical equation. (see appendix 9)

Alternative Dynamical Setting 1

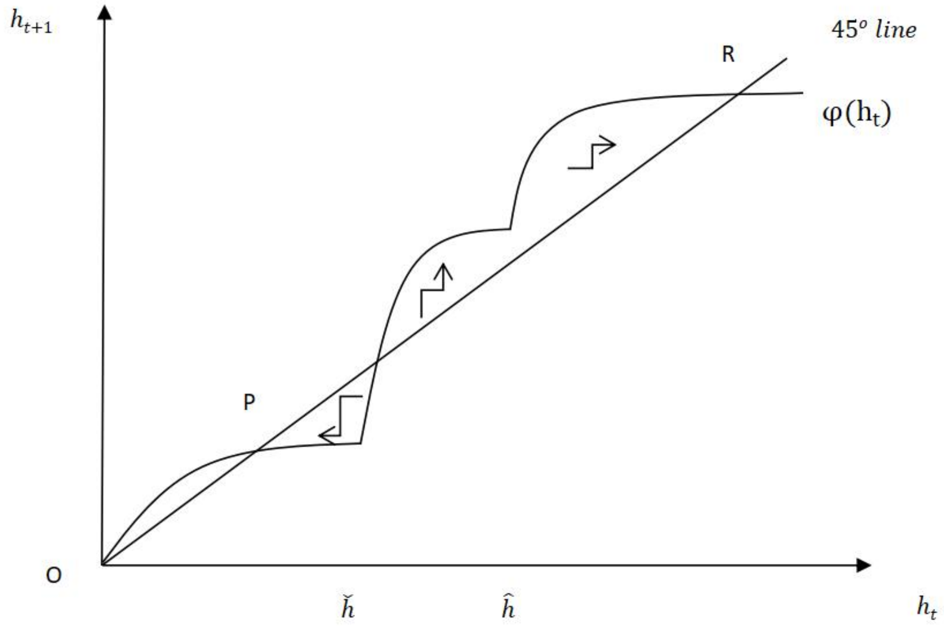

Figure - 5

Explanation of Figure - 5

- This Figure Holds if -

$$\frac{1}{A\gamma(1-b)e} \leq \frac{\overline{w}}{\underline{w}} \leq \frac{1+\gamma}{A\gamma(1-b)e}$$

- Here, the economy converges to long-run equilibria where the population is divided into two groups.

- The steady state value in this case is - $h^* = \frac{2A}{A(1-b)e\frac{\overline{w}}{\underline{w}} - \frac{1-\gamma}{\gamma}}$

Proposition - 6

- As $h^* = \frac{2A}{A(1-b)e\frac{\overline{w}}{\underline{w}} - \frac{1-\gamma}{\gamma}}$, higher $\frac{\overline{w}}{\underline{w}}$ indicates lower h^*.
- *Then higher the wage differential between the parents and child labour,*
  - *thereby inducing parents to substitute child education for child labour*
  - *higher the displacement effect (i.e., fall in* h^**)*
  - *higher the number of people coming out of the developmental trap.*

Alternative Dynamical Setting - 2

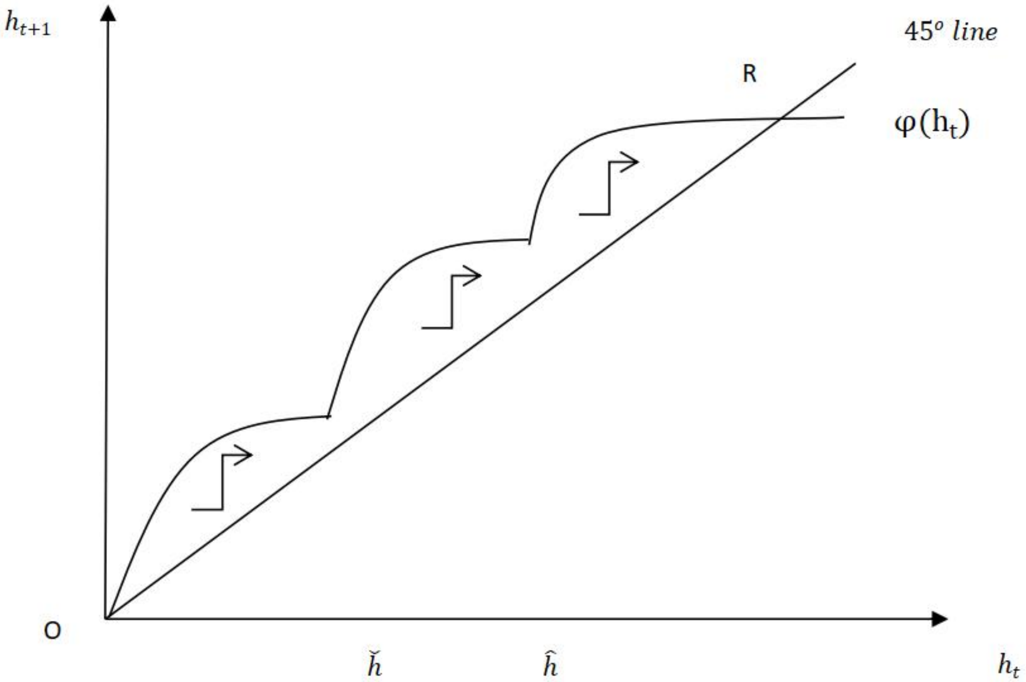

Figure - 6

Explanation of Figure - 6

- This Figure Holds if -

$$\frac{1}{A\gamma(1-b)e} \leq \frac{1+\gamma}{A\gamma(1-b)e} \leq \frac{\overline{w}}{\underline{w}}$$

- We have a unique stable equilibrium R.
- When the wage differential is low everyone converging towards good equilibrium.

Alternative Dynamical Setting - 3

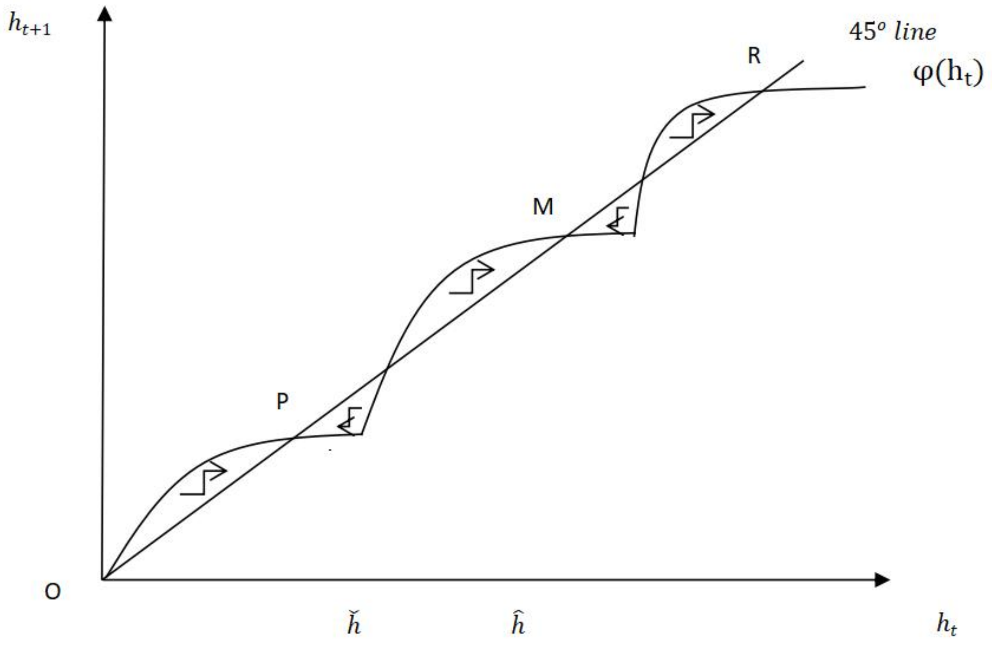

Figure - 7

Explanation of Figure - 7

- This Figure Holds if -

$$\frac{\overline{w}}{\underline{w}} \leq \frac{1}{A\gamma(1-b)e} \leq \frac{1+\gamma}{A\gamma(1-b)e}$$

- In this case we have multiple equilibria.
- Here, the economy is sub-divided into three groups, Poor, Middle and Rich.
- This indicates that if the wage differential is very high the economy can be sub-divided into three groups.

Proposition 8

- *If Parents Wage is High and Child Labour Wage is low, which implies wage differential is high, everyone, in the long run will reach to a good stable equilibrium where we will have zero child labour and full schooling.*
- *If Parents Wage is Low and Child Labour Wage is high, the economy will be subdivided into three groups in the long run, Poor, Middle and Rich.*

For UCTs

- Considering the first case as the general on where we have the following steady state values in case of UCT-

$$h^* = \frac{2A}{A(1-b)e\frac{\overline{w}}{\underline{w}} - \frac{1-\gamma}{\gamma}}$$

  – No change in the steady state value as compared to the general set up.

For GCCTs

- Considering the first case as the general on where we have the following steady state values in case of GCCT-

$$h^* = \frac{2A}{A(1-b)e\frac{\overline{w}}{\underline{w}} - \frac{1-\gamma}{\gamma}(1-\alpha)}$$

- The steady state value is decreasing with increase in α, similar to the basic model.
- Also, the steady state value under GCCTs is lower than UCT and form basic model.
- Thus, GCCTs is effective in reducing inequality.

For SCCTs

- Considering the first case as the general on where we have the following steady state values in case of SCCT-

$$h^* = \frac{2A}{A(1-b)e\frac{\overline{w}}{\underline{w}} - \frac{1-\gamma}{\gamma}}$$

- The steady state value remains unchanged as compared to the basic model.
- Thus, SCCTs is completely ineffective in reducing inequality via displacement effect.
- We received similar result in our basic model.

For PDIPs

- Considering the first case as the general on where we have the following steady state values in case of SCCT-

$$h^* = \frac{2A}{A(1-b)(e+\overline{e})\frac{\overline{w}}{\underline{w}} - \frac{1-\gamma}{\gamma}}$$

- Higher the value of $\overline{e}$, lower the h^*
- Indicating higher the displacement effect, i.e.,
- Fall in inequality resembling with the benchmark model.

Emergence of New “Middle” Group

- A new middle group emerges with this modified version of human capital accumulation,
- in case, when the wage differential is low which could be the case when the child labour wage is high and (or) parental wage is low.
- Thus, adding parents human capital in the child’s human capital generation function, we have got a new middle group for the case when the wage differential is low.

06

Conclusion

Key Insights

- Parents with lower human capital prefer higher fertility, while those with higher human capital focus on quality over quantity.
- Child labour wages increase fertility but reduce schooling levels.
- Low wage differentials create a poverty trap, keeping the poor disadvantaged while the rich progress.
- High wage differentials (high adult wages, low child wages) lead to universal schooling and eliminate child labour.

Government Initiatives

- Cash Transfer Programs:
  - UCTs → Increase fertility but do not impact schooling.
  - GCCTs → Lower fertility in higher-income groups, boost education, and reduce inequality.
  - SCCTs → Minimal impact on poverty traps but increase inequality.
- Population disincentive policies reduce inequality by lowering fertility rates, especially among low-income groups.

Extention

- Incorporating parental human capital highlights a critical dynamic:
  - high parental wages and low child labour wages result in a stable equilibrium with no child labour,
  - whereas low parental wages and high child labour wages create a stratified society with distinct poor, middle, and rich groups.
    - Emergence of New “Middle” group.
  - These results underscore the robustness of the findings within the study’s framework.

Limitations

- One of the most important limitation of our model is that we have not considered the production side so far and including the production side can change the result.
- In this study, we are assuming that the government is funded by external support when implementing the CT programs.
- It's important to examine whether a policy financed by debt leads to a higher or lower growth rate compared to one financed by external aid.

Thank You!

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